WHY DO WE NEED INSTANTONS IN STRANGENESS HADRON PHYSICS?

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Roles of instantons in strangeness hadron physics are discussed. After introducing general features of instantons, hadron spectroscopy under the influence of instanton-quark effective interactions is discussed. Emphases are on the H dibaryon, spin-orbit splittings of P-wave baryons and baryon-baryon interactions and spectrum of the pentaquarks.

1. Introduction

Strangeness nuclear physics has reached a new era, which requires elevation of phenomenological analyses of hypernuclear phenomena to a deeper understanding of the fundamental interactions from QCD viewpoint. Recent coordinated efforts in experimental and theoretical researches revealed several interesting features of hyperon-nucleon and hyperon-hyperon interactions. For instance, it is established by now that the spin-orbit interaction of Λ in nuclear medium is very weak, suggesting cancellation of two types of spin-orbit interactions, spin-flavor symmetric and antisymmetric ones. Discovery of pentaquarks triggers revisiting inter-quark correlations, such as diquarks and triquarks. It is urgent to establish foundations of such possibilities directly from QCD.

In this talk, I would like to review a new type of quark-quark interaction mediated by instantons in the QCD vacuum. The instanton idea is not new, but its applications to hadron spectroscopy and to hadronic interactions are being developed in these years. I concentrate on the instanton induced interactions among quarks, and stress that the strangeness plays an important role as the third flavor for the quark dynamics under the instanton induced interactions.

In sect. 2, instanton and instanton induced interaction are introduced in the context of hadron physics. In sect. 3, we discuss consequences of

the instanton induced interaction in the hadron spectroscopy and hadronic interactions. In sect. 4, we consider effects of instantons on the spectrum of recently discovered pentaquark baryon Θ^+ and its siblings. A conclusion is given in sect. 5.

2. Instanton

Instanton is a localized solution of the gluon equation of motion of QCD in the 4-dimensional Euclidean space, $D_\mu F_{\mu\nu} = 0$.[1] The solution also satisfies the self-duality (or anti-self-duality) relation

$$F^a_{\mu\nu} = \pm\tilde{F}^a_{\mu\nu} \equiv \pm\frac{1}{2}\epsilon_{\mu\nu\sigma\rho}F^a_{\sigma\rho}, \tag{1}$$

which guarantees that it satisfies the equation of motion because of the Bianchi identity, $D_\mu\tilde{F}_{\mu\nu} = 0$. An important feature of the instanton solution is its nontrivial topology, which is seen from non-zero topological charge (or winding number) defined by

$$\nu = \frac{g^2}{32\pi^2}\int F^a_{\mu\nu}\tilde{F}^a_{\mu\nu}\, d^4x \tag{2}$$

This ν takes an integer for localized $F_{\mu\nu}$ and represents the winding number associated with the homotopy group, $\pi_3(SU(2)) = Z$. The above relations leads to

$$S_E = \frac{1}{4}\int F^a_{\mu\nu}F^a_{\mu\nu}\, d^4x = \frac{8\pi^2}{g^2}|\nu|. \tag{3}$$

The instanton (anti-instanton) is a solution with $\nu = 1$ ($\nu = -1$) with a finite action density localized in the 4-dimensional space-time. One of the roles of the instanton in QCD is that it mediates quantum tunneling connecting two QCD vacua with different topologies. Indeed, the tunneling amplitude in the semi-classical limit is given by the instanton action, $T \sim e^{-S_E} \sim \exp(-\frac{8\pi^2}{g^2}|\nu|)$.

When quarks are introduced in QCD, there appears a new role of the instanton. The instanton acquires localized zero modes of light quarks, that is, $\lambda = 0$ solutions of the eigenvalue equation,[2]

$$i\not{D}q(x) = \lambda q(x). \tag{4}$$

It is easily seen that the right-chiral (R) and left-chiral (L) modes are paired with the opposite sign of λ for $\lambda \neq 0$,

$$i\not{D}\gamma^5 q(x) = -i\gamma^5\not{D}q(x) = -\lambda\gamma^5 q(x), \tag{5}$$

while for $\lambda = 0$, there exists an isolated mode with a definite chirality,

$$q_{\lambda=0} = \pm\gamma^5 q_0 \quad \text{for } \nu = \pm 1. \tag{6}$$

One sees that the coupling of the quark zero mode with the instanton breaks chiral symmetry,

Chiral symmetry of the QCD Lagrangian with N_f massless quarks is $U(N_f)_L \times U(N_f)_R$. By breaking this symmetry down to $U_f(N_f)$, we expect to have N_f^2 $(= 9$ for $N_f = 3)$ Nambu-Goldstone bosons. In reality, one of the nine NG bosons, η', is too heavy so that the $U_A(1)$ symmetry must be broken. It is realized by axial anomaly given by

$$\partial_\mu J_A^{\mu 0} = 2im_q \bar{q}\gamma^5 q + \frac{\alpha_s}{2\pi} N_f \mathrm{Tr}[F_{\mu\nu}\tilde{F}^{\mu\nu}]. \tag{7}$$

The anomaly term is proportional to the topological charge density of the instanton, which indicates that the QCD vacuum with instantons can describe the anomaly effect observed in the meson spectrum.

This role of the instanton was pointed out by 't Hooft,[2] who derived an effective interaction induced by the coupling of an instanton and light quarks via their zero modes. The strength of the interaction depends on the instanton density, which can be estimated from the gluon condensate of the QCD vacuum to be about 1 instanton/fm^4. The effective interaction, called instanton induced interaction (III), can be expressed as

$$L = \int d\rho\, n(\rho) \left[\prod_i \left(m_i\rho - \frac{3\pi^2\rho^3}{4} \bar{q}_R^i q_L^i \right) + (\text{tensor terms}) \right] + \ (\text{h.c.}) \tag{8}$$

where ρ is the size of the instanton and $n(\rho)$ is the instanton density.

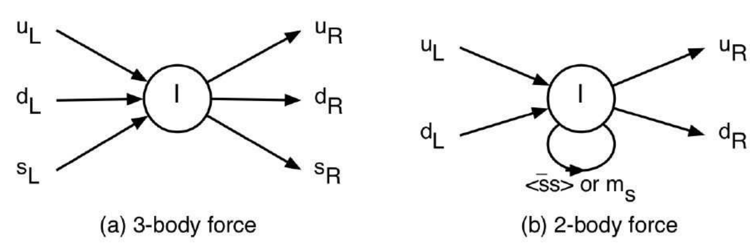

Figure 1. Instanton induced interactions: (a) Three-body term, and (b) Two-body term.

When the effective interaction is applied to the system of quarks, it can

be expressed conveniently as inter-quark potentials. For $N_f = 3$, we find[3]

$$V^{(3)}_{\mathrm{III}} = V^{(3)} \sum_{(ijk)} \mathbb{A}^f \left[1 - \frac{1}{7}(\vec{\sigma}_i \cdot \vec{\sigma}_j + \vec{\sigma}_j \cdot \vec{\sigma}_k + \vec{\sigma}_k \cdot \vec{\sigma}_i) \right] \delta(\vec{r}_{ij})\delta(\vec{r}_{jk})$$
$$V^{(2)}_{\mathrm{III}} = V^{(2)} \sum_{(ij)} \mathbb{A}^f \left[1 - \frac{1}{5}\vec{\sigma}_i \cdot \vec{\sigma}_j \right] \delta(\vec{r}_{ij}) \qquad (9)$$

where $V^{(3)}_{\mathrm{III}}$ is a three-body force (6-quark vertex) and is repulsive, while $V^{(2)}_{\mathrm{III}}$ is a two-body term, that is mostly attractive. (Fig. 1) The latter is generated from the three-body term by contracting a pair of quark and antiquark into quark condensate or in the quark mass term. In the above equations, $\mathbb{A}^f$ denotes antisymmetrization of flavor indices of the (ijk) or (ij) quarks. Then the interactions are nonzero only for flavor antisymmetric states of quarks, for instance, for $[ud](I = 0)$, and for $[uds]$ (flavor singlet). This flavor dependence plays an important role when they are applied to hadron spectroscopy.

Another important feature of III is that it mixes quark flavors in $\bar{q}q$ systems. It is easily seen that the III converts $\bar{u}u$ into $\bar{d}d$ or $\bar{s}s$. This provides an explicit mechanism to make the flavor singlet meson η_1 heavier than flavor octet mesons, π, K and η_8, and thus explains the $U_A(1)$ symmetry breaking. Mixing III with appropriate flavor SU(3) breaking interactions, it was shown that pseudoscalar meson spectrum can be explained fairly well.

3. Hadrons with instanton induced interaction

Lattice QCD suggests[4] that the QCD vacuum indeed contains instantons, whose density is consistent with the gluon condensate expected from QCD sum rules. It was also shown that chiral symmetry is dynamically broken in the instanton vacuum. Then massless quarks are transformed into constituent quarks, which acquire mass as a function of momentum. In recent developments, III is pointed out to play a major role in diquark correlations in hadrons and quark (hadron) matter. Antisymmetric ud diquark with $I = 0$ (or flavor $\bar{3}$), $J^\pi = 0^+$ and color $\bar{3}$ is strongly favored by III. Such diquark correlations may induce phase transition of the QCD vacuum at high density into a color superconducting phase.

Hyperfine interaction and H dibaryon

The main role of III in the low-energy hadron spectroscopy is given by its spin dependence. In the conventional quark model, splittings of the

ground state octet and decuplet baryons are supposed to come from color-magnetic terms of one-gluon exchange (OgE) interaction between quarks. Indeed, if we choose the strength of the OgE interaction to be consistent with the $N-\Delta$ mass splitting (exp. 300 MeV), then it can explain the whole octet and decuplet spectra very well. It is, however, too strong in the sense that the coupling strength, α_s, thus determined is larger than one, and perturbative expansion is not justified.

Another problem of OgE is seen in the spectrum of double-strange dibaryon H ($S=-2$, $B=2$).[5] A deeply bound H-dibaryon was predicted, that is, the biding energy of more than 100 MeV. A typical estimate of the H dibaryon mass in the constituent quark model gives

$$M_H = \sum m_q + \langle V_{cm}\rangle_H = 360 \times 4 + 540 \times 2 - 450 \sim 2070 \text{ MeV}, \quad (10)$$

which is about 150 MeV above the $\Lambda\Lambda$ threshold ($\sim$ 2230 MeV). Unfortunately, 20-year effort of searching the H dibaryon was not successful.

An alternative to OgE is the two-body part of III, which favors lower total spin. In 1989, we proposed a picture in which a part of the hyperfine interaction is shared by III from OgE.[3] This picture works because the ground state baryon spectrum is equally well reproduced by III, because its spin dependence is almost identical to that of OgE.[6] Even the required mass dependence of the hyperfine splitting is explained as the effect of the strange quark mass on the instanton vacuum. In the III+OgE quark model, the sum of the hyperfine interaction is fixed by the splitting of the octet and decuplet baryons. We introduce a parameter, p_{III}, that represents the portion of the hyperfine splitting given by III. For p_{III} is zero, the $N-\Delta$ mass difference comes solely from OgE, while III is responsible entirely for $p_{III}=1$.

The mixing strength may be determined in the quark model using the $U_A(1)$ breaking in the pseudoscalar meson spectrum, that is, the $\eta-\eta'$ mass difference. A typical magnitude is about $p_{III} \sim 40\%$. Then α_s is reduced by a factor 0.6, which is an attractive feature of this model.

It should be noted that the three-body term does not change the above picture, because it is effective only on flavor-singlet $u-d-s$ system. III that is consistent with the $N-\Delta$ mass splitting also explains the ground state baryon spectrum in the same quality as OgE. Flavor singlet baryon, Λ^*, is then the only place to see this effect in the ordinary meson-baryon spectrum. Even in the Λ^*, it is highly suppressed at $L=1$ relative motion because the interaction is strong only at short distance determined by the size of the instanton ($\sim 0,3$ fm).

In contrast, the three-body III plays a key role in the H dibaryon.[7] Note that H is a flavor singlet 6-quark system, and it contains multiple flavor-singlet $u-d-s$ components. It is found that although the two-body III gives some attraction, the three-body term gives strong repulsion to push the H dibaryon above the $\Lambda\Lambda$ threshold.

Spin-orbit interactions

Spin-orbit interactions of quarks and baryons provide an important clue to III. It is well known that LS splittings of N^* baryons are unexpectedly small. This suggests that the LS force among quarks is weak. However, when we consider NN interaction in terms of quark substructure of the nucleon, it requires much stronger LS force for quarks. Recent studies revealed that the one-body LS force on Λ in hypernuclei is much weaker than that for N. This can be explained by cancellation of two types of LS force, that is, flavor-symmetric (SLS) and antisymmetric (ALS) LS forces,

$$\begin{aligned} V_{SLS} &= V^0_{SLS}\,(\vec{\sigma}_N + \vec{\sigma}_\Lambda)\cdot\vec{L}, \\ V_{ALS} &= V^0_{ALS}\,(\vec{\sigma}_N - \vec{\sigma}_\Lambda)\cdot\vec{L}. \end{aligned} \tag{11}$$

ALS is extremely small for the NN interaction because it necessarily breaks isospin symmetry. In contrast, it can be as strong as SLS in the YN and YY interactions. Indeed, supposing $V^0_{SLS} \sim V^0_{ALS}$ in the ΛN LS potentials, the one-body LS force on Λ in hypernuclei is strongly suppressed due to the cancellation of SLS and ALS.

Takeuchi performed the calculation of the LS force in the instanton induced interaction and found that its flavor-dependence is different from the LS force in OgE.[8] She pointed out that the LS splittings of the P-wave N^* baryons are reduced significantly by mixing III (Fig. 2), while the NN and YN LS forces remain strong. In the quark cluster model (QCM) calculation with III, it is found that the antisymmetric LS force V_{ALS} between Λ and N is strong, so that it strongly reduces the LS force for Λ. It is also pointed out that coupling of $N\Sigma$ plays an important role in such a calculation.

4. Roles of Instantons in Pentaquarks

Discovery of Θ^+[9] generated a lot of interest in the pentaquark states. The pentaquark is a baryon which cannot be composed of three constituent quarks. Because Θ^+ is supposed to be a $S=+1$ baryon resonance, its minimal quark configuration is $uudd\bar{s}$, thus a pentaquark.

It was confirmed by several other groups later, but some negative results have been reported from high energy e^+e^- and proton induced production

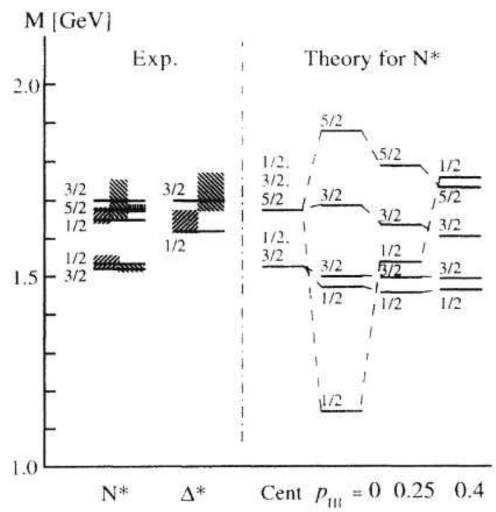

Figure 2. P-wave N^* spectrum in III compared with experiments.

experiments. Its width seems to be very small (~ 1 MeV or less), which is a great mystery to be solved. The quantum numbers other than the baryon number, strangeness and isospin ($I = 0$) are not known.

A naive constituent quark model may predict a ground state with negative parity and spin 1/2, while some quark models predict a possibly narrow negative parity state, which decays only to KN P-wave states.[10] Dynamical calculations of the pentaquark systems in the quark model are yet to be seen, but in general, it is not easy to bring a five-quark object down to 1.54 GeV of mass.

Recently, an analysis of the contribution of the III for Θ^+ in the MIT bag model was carried out by Shinozaki et al.[11] One important mechanism of III is that the two-body part of III is strongly attractive. Furthermore, as its leading term is independent of spin, the attraction grows with the number of qq bonds in the system, which is ten for 5-quark systems and three for 3-quark systems. The three-body part of III is repulsive, which amounts to 50 MeV. This is smaller than the repulsion in the H dibaryon, because Θ^+ contains only one strange (anti)quark. In the end, we find that the instanton gives moderate attraction (~ 100 MeV) to the negative parity pentaquark (Fig. 3). It is also found that the spin 3/2 partner of Θ^+ is strongly affected by III, while $S = -2$ Ξ^{--}, which is another possible genuine pentaquark

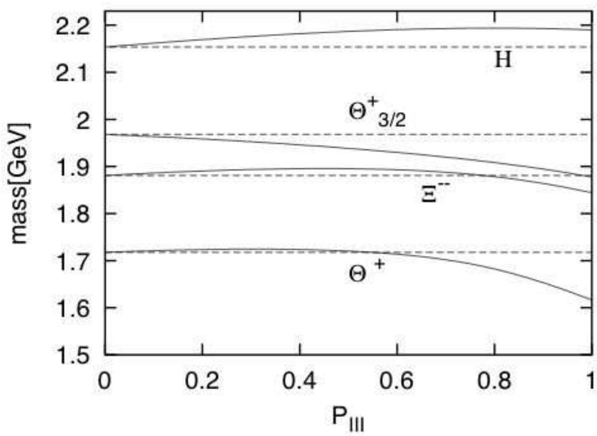

Figure 3. Masses of pentaquark baryons and the H dibaryon as functions of p_{III} calculated in the bag model.

baryon, is insensitive to III. III also gives distinctive effects on the positive-parity pentaquark state, because the component favored by III is different from that by OgE.

5. Conclusion

The instanton represents nonperturbative effects of QCD in hadron spectroscopy and interactions. Instanton induced interaction can be seen unambiguously in several key quantities, such as $\eta - \eta'$ mass difference, H dibaryon and maybe, pentaquarks. III may also explain the spin-orbit forces in baryon spectrum and B-B interactions. Pentaquarks have attraction from III.

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